QEG - Oral Exam 2018

Second question

Nuclear Science & Engineering Department
MIT Cambridge, MA 02139, USA

For each of the questions below, make sure you define all the quantities, physical systems, acronyms, jargon, etc., that are given before tackling the question.

Problem 1:

a) Describe Ramsey interferometry with a single atom and how it can be used to measure frequency.

b) Now consider that you have N atoms . You prepare the first atom in a superposition state (e.g. $(|0\rangle + |1\rangle)\sqrt{2})$ and then you want to create a GHZ state of all the atoms. What simple two-qubit quantum gate could you use (recursively) to prepare this entangled state?

c) What would be the result of using the GHZ state for Ramsey interferometry?
Assume you first create the GHZ state as above, then let the atoms to freely evolve and finally invert the operations done to create the GHZ state before measuring atom # 1.

d) Assuming the dynamics is unitary, what is the quantum limit of the frequency measurement sensitivity in this interferometer, for a given time T?

Problem 2:

a) What is the quantum limit of the frequency measurement sensitivity in the presence of decoherence?
Assume that each atom is coupled to its own environment and that the resulting dephasing rate is identical for all atoms (i.e. single-particle off-diagonal density matrix elements would e.g., decay as $\langle 0|\rho(T)|1\rangle = \exp(-\gamma T)\langle 0|\rho(0)|1\rangle$). Assume also that decoherence is only relevant during the free evolution period. In particular, determine the minimum sensitivity optimized with respect to interaction time T.

b) [If there is time]
Compare this optimal limit with the corresponding result for uncorrelated atoms in the usual Ramsey setup.

c) What error correction strategies could be used to protect the atoms from the noise? Are they compatible with the measurement task?