Simplified aerodynamic heating of rockets

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Introduction

When a rocket travels through the air, the surface parts get heated by ”the friction with the air”. In a naivistic view, the air molecules may be thought of as small particles that hits the rocket unelastically, whereby the particle kinetic energy is converted to heat. This naivistic description is may help the intuitive understanding of aerodynamic heating, but it is not in any way a realistic model of the real phenomena for three reasons:

1. The air flows ***around*** the rocket, and the temperature of the airflow depends on local conditions. The surface of the rocket does not reach ***a*** temperature, but rather a temperature distribution.
2. The aerodynamic heating does not really originate from friction, but rather from heat transfer from the heated airflow to the rocket body surface, and this heat transfer from the airflow to the surface of the rocket is not perfect.
3. The surface heating is a dynamic process, where the heat capacity of the surface introduces a lag in the temperature of the surface with respect to that of the flow.

In the following, a simplified surface temperature calculation based on a semi empirical method, is outlined. The method is based on [1], but extended to cover stagnation points and two dimensional flow.

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The basic method

The skin temperature of the rocket depends on the net heat transfer into the rocket surface, and the heat capacity of the rocket surface. Since this depends on local conditions, an exact solution of the temperature distribution requires a full three dimensional thermodynamic calculation of vast complexity. It is however reasonable to assume that the lateral (along the rocket body) temperature gradient is much smaller than the transversal (through the skin) gradient. It is thus reasonable to neglect the lateral heat flow completely. In this way the temperature at any station along the rocket surface can be calculated independently, and these calculations can then be combined into a full temperature profile.

Note that the description of the basic method is more or less a rewrite of the corresponding description in [1].

Heat balance

The heat flux (energy pr. unit area pr. unit time) from the surrounding air into the skin of the rocket body may be expressed as:

$$\dot{Q}_1 = h(T_B - T_S)$$

Where T_B is the temperature of the boundary layer and T_S is the temperature of the surface of the rocket. The heat transfer coefficient h has been determined experimentally by Eber [2].

It is assumed that Q_1 is the dominant contributor of heat flow into the skin and other sources are therefore neglected.

The rocket looses some heat however due to radiation. The radiation loss may be expressed as:

$$\dot{Q}_2 = \epsilon \sigma T_S{}^4$$

where $\sigma = 5.67*10^{-8}$ W/m^2/K^4 is the Stefan-Boltzmann constant and ϵ is the emissivity of the skin.

During a time interval dt, the heat balance for the skin temperature can be written as:

$$G\,dT_S = dt(\dot{Q}_1 - \dot{Q}_2) \Leftrightarrow$$

$$G\frac{dT_S}{dt} + h\,T_S + \epsilon \sigma T_S{}^4 = h\,T_B$$

Where G is the "skin heating capacity" [J/m^2/K], the product of the specific heat of the skin material, the density of the skin material and the thickness of the material:

$$G = c \rho \tau$$

For an object that flies at constant velocity, the term $dT_S / dt \rightarrow 0$ over time, and the skin reaches an equilibrium temperature determined by:

$$hT_{S,eq} + \epsilon \sigma T_{S,eq}{}^4 = hT_B \Leftrightarrow$$

$$h = \frac{\epsilon \sigma T_{S,eq}{}^4}{(T_B - T_{S,eq})}$$

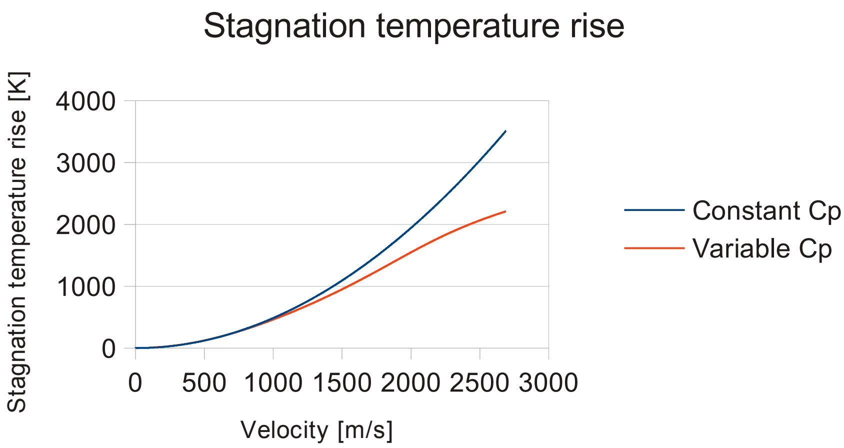
Boundary layer temperature

When the flow is isentropically brought to rest, the temperature of the flow will be the so called stagnation temperature. This mimics the situation at the nose cone apex, as for the rest of the rocket body, the flow will be brought partially to rest by means of skin friction, and the boundary layer temperature will be somewhat lower than the stagnation temperature.

The stagnation temperature (also known as the *total* temperature) T_{st} relates to the free stream temperature T_{fs} and velocity V_{fs} in the following way:

$$T_{st} = T_{fs} + \frac{V_{fs}^2}{2C_p}$$

assuming that the specific heat capacity at constant pressure - C_p - is independent of temperature. As this is not entirely true, a slight correction may be necessary at high velocities.



The boundary layer temperature relates to the stagnation temperature in the following way (by definition):

$$K = \frac{T_B - T_{fs}}{T_{st} - T_{fs}}$$

K is known as the temperature recovery factor. K was experimentally determined by Eber [2] for conical bodies. Eber found that for cones with vertex angles of 20 – 50 degrees, a value of K=0.89 may be used over the entire velocity range. The boundary layer temperature rise may then be expressed as:

$$(T_B - T_{fs}) = K(T_{st} - T_{fs}) = K\left(\frac{V_{fs}^2}{2C_p}\right)$$

Eber empirically modeled the heat transfer as:

$$h = (0.0071 + 0.0154\sqrt{\beta})\frac{k}{l}R^{0.8}$$

Here, k is the thermal conductivity of air, *l* is a characteristic length - defined by Eber as the length of the cone, R is the Reynolds number and β is the vertex angle of the cone (in radians).

Substituting for the Reynolds number yields:

$$h = (0.0071 + 0.0154\sqrt{\beta}) \frac{1}{l^{0.2}} (\rho_{fs} u_{fs})^{0.8} \frac{k}{\mu^{0.8}}$$

where μ is the dynamic viscosity of air.

Composite skin temperature profile

The simplified method may be used to calculate the skin temperature at several stations of a nose cone, then combining them to a full temperature profile. This is straightforward, assuming that each station is located an a cone surface that tangentiates the nose cone at the station. The only question is which reference length to use. According to [1], there is no general consensus of what *l* is, and sometime half the length of the cone is used.

For composite temperature calculations, the somewhat conservative approach of using the surface length of the tangent cone, from its projected apex to the station of interest will be suggested for *l* as suggested in [3].

Limitations of the method

There are several limitations of this method. The applicable altitudes may not exceed 130000 ft (43 km). Changes in the atmosphere properties may be significant at higher altitudes, but as ρ_{fs} then decays rapidly, extrapolation to higher altitudes may be justified.

Also the method is based on measurements within a limited range of Reynolds numbers ($2*10^5$ - $2*10^6$), and extrapolation outside this range may be required during application of this method.

The emissivity ϵ is 1 for a black body, but the value for real rocket skin may not be known. In [1], a value of $\epsilon = 0.4$ is assumed, but it is also noted that radiation losses are small compared to the aerodynamic heating and the use of a fixed ϵ is justified, even if the value is not fully correct.

Furthermore, the method implicitly assumes that the heat conduction within the skin material is instant, so that the temperature may be considered uniform within the skin. Although the heat conduction may be much better for solid materials – and especially for metals - than for gases, the difference may not be so prominent for composite materials. This may drive the need for introducing a purely empirical ”shortening factor” basing the calculation on a virtual skin thickness, smaller than the real skin thickness. For good conducting materials like aluminum, the ”shortening factor” may be almost 1, but for composite materials, a smaller value may be required. This leads to another problem: Composite materials will start to ablate when exposed to high temperatures. The ablation process is often followed by charring of the material thus providing the combination of temperature reduction, higher emissivity and higher temperature working range. Naturally, ablation will affect the validity of the skin temperature calculations.

Finally, the outlined method does not cover the case of $\beta \rightarrow \pi/2$ and it does not directly cover the two dimensional flow around the fins.

Extension of the method

The most severe restriction of the basic method is that it is limited to $20^o < \beta < 50^o$. Simply extending the range down to 0^o is likely to overestimate the skin temperature, while extending to 90^o is likely to underestimate the skin temperature. Overestimating the skin temperature at small values of β is not a serious problem as this typically corresponds to a station far from the apex, where the skin temperature is less critical. The real issue, from a designers point of view, is the temperature of the apex region, where the flow is stagnant. In this region, one can use the stagnation temperature as an upper bound, but this may be too conservative.

Another issue from the designers point of view is the heating of the fins, especially at the leading edges, where the wall thickness tend to be small.

In the general case, only two things have to be determined in order to solve the aerodynamic heating equation

$$G\frac{dT_S}{dt}+hT_S+\epsilon\sigma T_S^4=hT_B$$

The recovery factor K must be determined for the actual flow situation, and so must the heat transfer coefficient. The skin capacity G has the same definition regardless of the skin is that of a cone, a fin or a stagnation point.

Recovery factor

At the nose cone apex, the conditions must resemble that of a stagnation point, and the boundary layer temperature T_B is assumed to equal stagnation temperature T_{st} hence K=1.

For cone angles approaching zero, the flow may be assumed to pass freely, hence the recovery must approach zero.

Although both assumptions seem fair, they are oversimplifications. At the apex – or even in the case of a flat plate – the air flow moves around the obstacle, and despite the ”stagnant like” conditions, the recovery factor remains below 1. At cylindrical conditions ($\beta = 0^o$), there is still a boundary layer with a temperature gradient. The recovery factor has some correlation with the skin friction, and can not be assumed to be zero.

In general, the recovery factor is a function of the Reynolds number, the Mach number and the Prandtl number of the flow situation. For air, a Prandtl number of 0.7 may be assumed as representative, thus reducing the recovery factor to be a function only of local Mach and Reynolds numbers.

For flow over a flat plate, corresponding to the fin of a rocket, the Mach number has very little influence on the recovery factor, and the Reynolds only separates the cases of laminar and turbulent flow:

$$K=\begin{Bmatrix}\sqrt[3]{Pr}\sim 0.89\ \mathit{for\ turbulent\ flow}\\ \sqrt{Pr}\sim 0.84\ \mathit{for\ laminar\ flow}\end{Bmatrix}$$

At the fins, the flow will be turbulent in any realistic case, and a value of K=0.89 may be assumed.

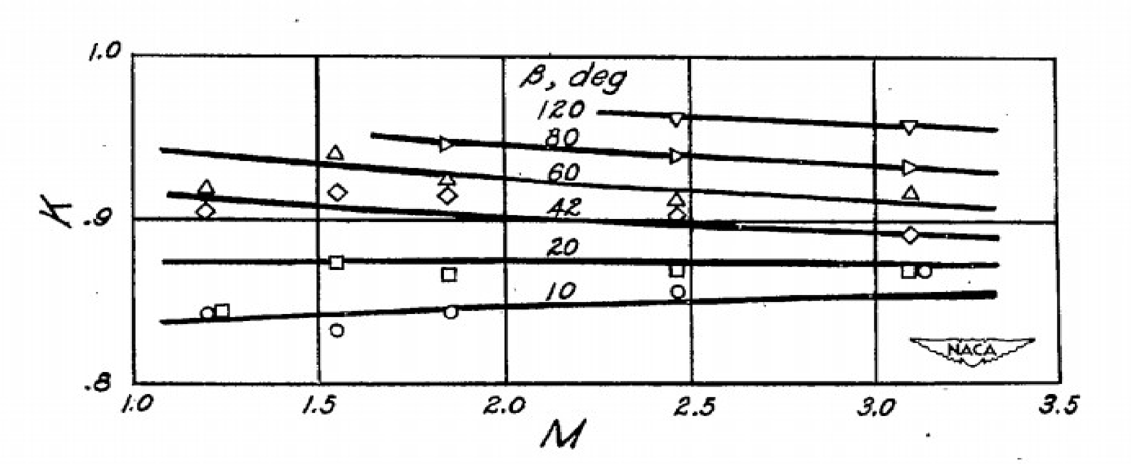
The condition separating laminar or turbulent flow is:

$$Re_{lok} = \begin{cases} \frac{\rho u L}{\mu} = \frac{u L}{\nu} > 10^6 \text{ for turbulent flow} \\ \frac{\rho u L}{\mu} = \frac{u L}{\nu} < 10^6 \text{ for laminar flow} \end{cases}$$

Where μ and ν are the dynamic respectively the kinematic viscosity of the air and ρ is the density of the air at the distance L from the nosecone. There is some controversy about the number 10^6 as transition may occur unpredictatly at any Reynolds number in that order.

For the flow around the nose cone, the story is slightly different. For small values of β and stations far from the apex, the flat plate model may be an acceptable approximation. Ebers measurements of recovery factors on cones show a systematic relation with β and a slight negligible variation with the Mach number. Knowing that Eber made his measurements around the critical Reynolds Number, it is interesting to see that the measurements on cones with low values of β low Mach numbers approach K=0.84, while they approach K=0.89 at higher Mach (and Reynolds) numbers. Since the difference is only about 5%, it is justifiable to ignore the difference between turbulent an laminar flow and simply assume that the universal value of K=0.89 is valid for β < 40 deg. For larger values of β there is a slight correction of 10% pr degree:

$$K = \begin{cases} 0.89 & \text{for } \beta < 40\,deg \\ 0.89 + 0.001(\beta - 40\text{deg}) & \text{for } \beta \geq 40\,deg \end{cases}$$



Ebers Recovery factors for cones. From [3]

Heat transfer coefficient

The heat transfer coefficient may be determined from steady state measurements, if the boundary layer temperature and emissivity is known.

$$h = \frac{\epsilon \sigma T_{S,eq}^{\ 4}}{(T_B - T_{S,eq})}$$

If the radiation loss is ignored, there is no heat transfer between the skin and the flow, and the temperature on the skin reaches the adiabatic stagnation flow temperature:

$$T_{ad}=\left(1+\frac{\gamma-1}{2}M_{fs}{}^{2}\right)T_{fs}$$

Where M_{fs} the free stream Mach number.

The adiabatic stagnation flow temperature thus represents the upper boundary for the skin temperature.

For rockets, the steady state conditions are unlikely to occur, and there will be a net flux of energy between between the skin surface and air flow. The heat transfer coefficient h relates to the Nusselt number in the following way:

$$Nu=\frac{hl}{k}$$

where the reference length *l* is a representative physical dimension of the obstacle.

Compared with Ebers empirical expression for h, an empirical expression for the Nusselt number for cones can be stated:

$$Nu=(0.0071+0.0154\sqrt{\beta})R_e^{0.8}$$ presumably for turbulent flow

For flat plates, the Nusselt Number may be expressed as [4]

$$Nu=\begin{Bmatrix} 0.664\sqrt{R_e}\sqrt[3]{Pr} \sim 0.591\sqrt{R_e} & \text{for laminar flow} \\ 0.037R_e^{0.8}\sqrt[3]{Pr} \sim 0.0329R_e^{0.8} & \text{for turbulent flow} \end{Bmatrix}$$

using the atmosphere properties at an ”average” temperature of

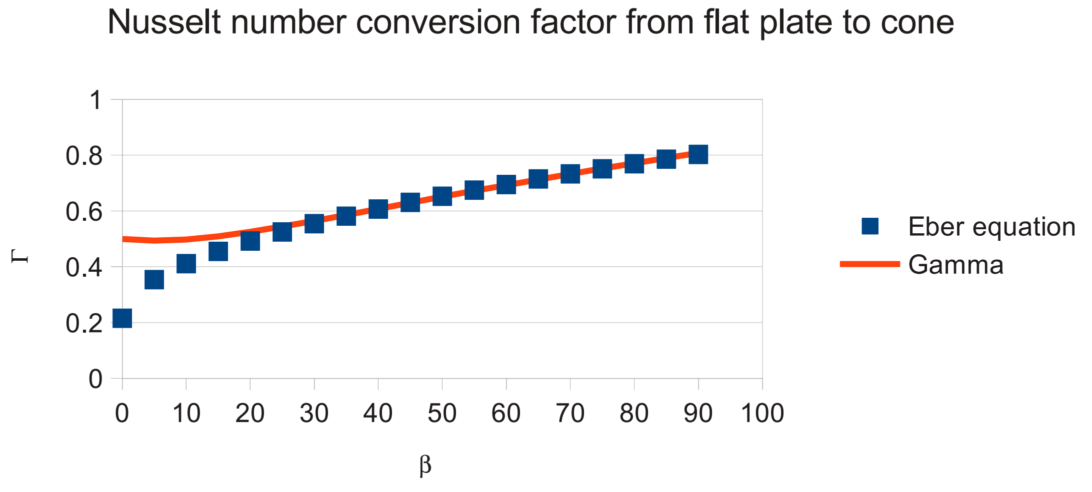
$$T_{avg}=T_{fs}+0.5(T_{skin}-T_{fs})+0.22(T_{ad}-T_{fs})$$

The expressions for cones and flat plates follows the same expression for turbulent flow, except for a scale factor. It may be convenient to merge the expressions into:

$$Nu=Nu_{flat\ plate}\Gamma_{shape}$$

$$\Gamma_{shape}=\begin{Bmatrix} 1 & \text{for flat plates} \\ \sqrt{0.0058\beta+0.13+0.12e^{-0.07\beta}} & \text{for cones} \end{Bmatrix}$$ (β in degrees)

The conversion factor for cones can be seen below. It is reasonable that the factor should limit at a factor of 0.5 at low values of β, which corresponds with Ebers model at its stated 20° lower range.



Even though it is not directly mentioned, Ebers model presumably covers the case of turbulent flow only. There is no reason to believe that the laminar flow Nusselt number model for flat plates should not apply also for cones, and the shape conversion should therefore apply for both flow regimes.

For the nose cone apex, the shape may be approximated with a sphere of diameter D. Here the stream conditions are considered to be subsonic, even in supersonic flow, as the flow will be subsonic behind the normal shock wave. The heat transfer coefficient may be expressed as [5]:

$$h=\frac{k_{sk}}{\sqrt{\nu_{sk}}}\left[\frac{Nu}{\sqrt{Re_{sk}}}\right]\sqrt{C}$$

Where

$$C=3\frac{u_0}{D}\left[1-0.252\,M_0{}^2-0.0175\,M_0{}^4\right]$$ For a sphere of diameter D

$$C=2\sqrt{2}\frac{u_0}{D}$$ For a wire of diameter D in cross flow

ν is the kinematic viscosity of air

u_0 is the free stream velocity [m/s]

k is the thermal conductivity of air

M_0 is the free stream Mach number if < 1. Otherwise it is the Mach number behind a normal shock.

Index sk indicates that properties are calculated at the skin wall temperature.

The property $\left[\frac{Nu}{\sqrt{Re_{sk}}}\right]$ is derived from charts in reference [5]:

$$\left[\frac{Nu}{\sqrt{Re_{sk}}}\right]=\left[0.464+6.86\cdot10^{-2}\left(\frac{T_{sk}}{T_0}\right)-8.54\cdot10^{-3}\left(\frac{T_{sk}}{T_o}\right)^2\right]\left[\frac{Pr}{0.8}\right]^{0.4}$$ For wire in crossflow $0 < T_{sk}/T_0 < 2$

$$\left[\frac{Nu}{\sqrt{Re_{sk}}}\right]=\left[0.640+6.67\cdot10^{-2}\left(\frac{T_{sk}}{T_0}\right)-8.17\cdot10^{-3}\left(\frac{T_{sk}}{T_o}\right)^2\right]\left[\frac{Pr}{0.8}\right]^{0.4}$$ For sphere $0 < T_{sk}/T_0 < 2$

Where T_0 is the free stream temperature if the free stream Mach number < 1. Otherwise it is the Temperature behind a normal shock.

Skin gradient

A drawback of the original method is that it is primarily aimed at metallic skin materials with good thermal conduction, justifying the assumption that the calculated skin temperature is present along the entire wall thickness of the skin. For composite materials, this assumption may not hold, due to the poorer heat conduction of the material. The method can be improved by allowing a temperature gradient within the skin.

If the exterior side of the skin is at a temperature of T_S and the interior side of the skin is at a temperature of T_i then there is an internal heat flux within the skin of

$$\dot{Q}_i=\frac{-\lambda}{\tau}(T_S-T_i)$$

The heat flux into the skin is

$$\dot{Q}=\dot{Q}_1-\dot{Q}_2\approx Q_1=h(T_B-T_S)$$

The assumption of uniform skin temperature is reasonable when the interior ”heat transfer” is much better than the heat transfer from the boundary layer to the skin:

$$\frac{\lambda}{h\tau}\gg 1$$

Now, assume that some amount of heat has entered the skin material. The has caused a temperature rise of ΔT_S at exterior side of the skin and of ΔT_i at the interior side of the skin. The skin temperature calculation assumes uniform temperature however, and will result in a value of

$$\Delta\hat{T}_S=\frac{\Delta T_S+\Delta T_i}{2}=\frac{\Delta T_S}{2}(1+x_\tau)$$

It is clear, that if the calculation of $\Delta\hat{T}_S$ had been done with half the wall thickness (or half thermal mass), it would reach the double value – or in general:

$$\hat{\tau}=\delta\tau\rightarrow\Delta\hat{T}_S=\frac{\Delta T_S}{2\delta}(1+x_\tau)$$

This means that $\Delta\hat{T}_S=\Delta T_S$ when

$$\delta=\frac{(1+x_\tau)}{2}$$

Aplying this ”shortening factor” δ to the wall thickness τ will compensate for a non uniform temperature distribution within the skin. It can be seen that δ =1 corresponds to the case of uniform temperature distribution while δ =1/2 corresponds to the case of very poor heat conduction.

On this basis, the use of a purely empirical shortening factor is suggested:

$$\delta=\frac{2-e^{\frac{-\lambda}{h\tau}}}{2}$$

Ablation

A simple ablation model can be added, by assuming that ablation happens at a known and constant temperature, T_{ablate} [K], and at a known and constant heat of ablation H_{ablate} [J/kg]. Furthermore it is assumed that ablation does not affect the boundary layer in any way.

In this case, the skin temperature does not exceed the ablation temperature, and at the ablation temperature all the heat flux into the skin is spent evaporating the material. As such, the wall thickness τ will reduce while the skin temperature is larger than the ablation temperature:

$$G\frac{dT_S}{dt}=h(T_B-T_S)-\epsilon\sigma T_S{}^4+H_{ablate}\rho\frac{d\tau}{dt}\qquad with\left\{\begin{array}{l}\frac{dT_S}{dt}=0\ for\ T_S\geq T_{ablate}\\ \frac{d\tau}{dt}=0\ for\ T_S<T_{ablate}\end{array}\right\}$$

It follows that

$$\frac{d\tau}{dt}=\begin{cases} \dfrac{h(T_B-T_S)-\epsilon\sigma T_S^{4}}{-H_{ablate}\rho} & for\ T_S \geq T_{ablate};\ h(T_B-T_S)-\epsilon\sigma T_S^{4} \geq 0 \\ 0 & for\ T_S < T_{ablate};\ h(T_B-T_S)-\epsilon\sigma T_S^{4} < 0 \end{cases}$$

Charring may be modeled by setting $\epsilon=1$.

Literature

1. NACA Technical Note No. 1725. ”Determination of transient skin temperature of conical bodies during short-time, high speed flight”. Hsu Lo, Langley Aeronautical Laboratory, October 1948.
2. ”Experimentelle Untersuchung der Bremstemperatur und des Wärmeüberganges an einfachen Körpern bei Überschallgeschwindikeit”. G. R. Eber, Peenemünde, November 1941.
3. NACA Technical Note 1724. ”A study of skin temperatures of conical bodies in supersonic flight”. Wilber B. Huston, Calvin N. Warfield, Anna Z. Stone. Langley Aeronautical Laboratory, October 1948.
4. ”Aerodynamic heating”. MECH 448 lecture slides. Queen's University, Department of Mechanical and Materials Engineering.
5. NACA Technical Note 3513. ”Heat transfer at the forward stagnation point of blunt bodies”. E. Reshotko and C. B. Cohen. Lewis Flight Propulsion Laboratory, July 1955.
6. ”Heat and Mass Transfer”. E. R. G. Eckert and R. M. Drake, Jr. McGraw-Hill Book Company, Inc., international student edition.
7. US STANDARD ATMOSPHERE SUPPLEMENTS, 1966 (60deg N, July)

Appendix

Atmosphere parameters

The local speed of sound may be expressed as:

$$c = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \quad \text{for an ideal gas}$$

γ is the *adiabatic index* = 1.4 for an ideal gas

ρ is the local density of air

P is the local pressure

T is the local *absolute* temperature

M is the molar mass of the air = 0.0289645 kg/mol

R is the universal gas constant = 8.3145 J/mol/K

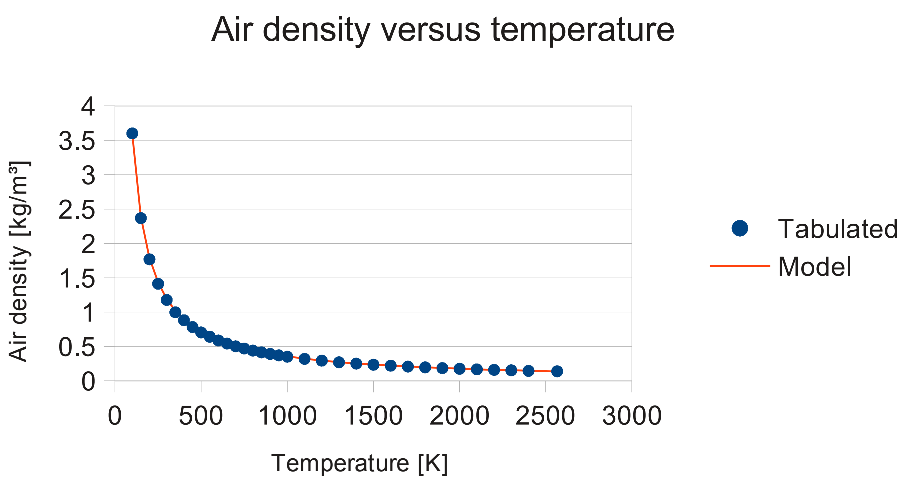
If the speed of sound c_{ref} is known at some reference temperature T_{ref} then this can be used to calculate c at temperature T:

$$c(T) = c_{ref} \sqrt{\frac{T}{T_{ref}}}$$

Then Mach number is the velocity of the flow divided by the speed of sound:

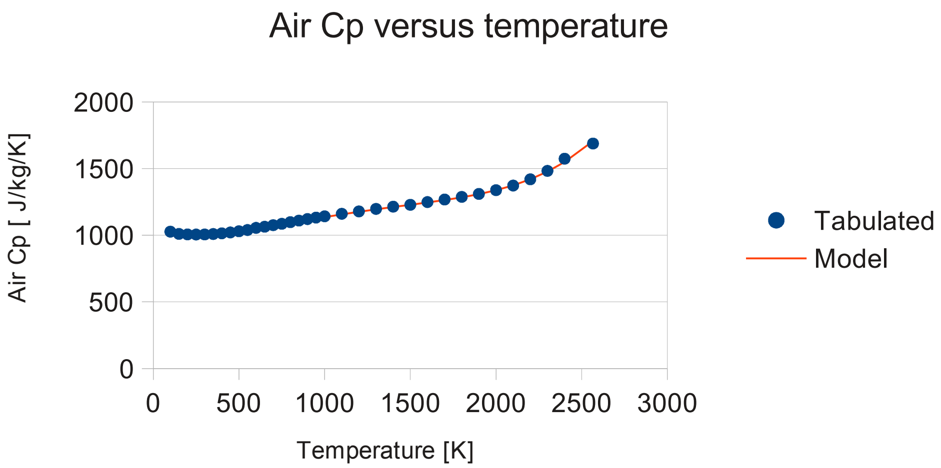
$$M = \frac{u}{c}$$

The following properties have been curve fitted from tabulated values of reference [6].

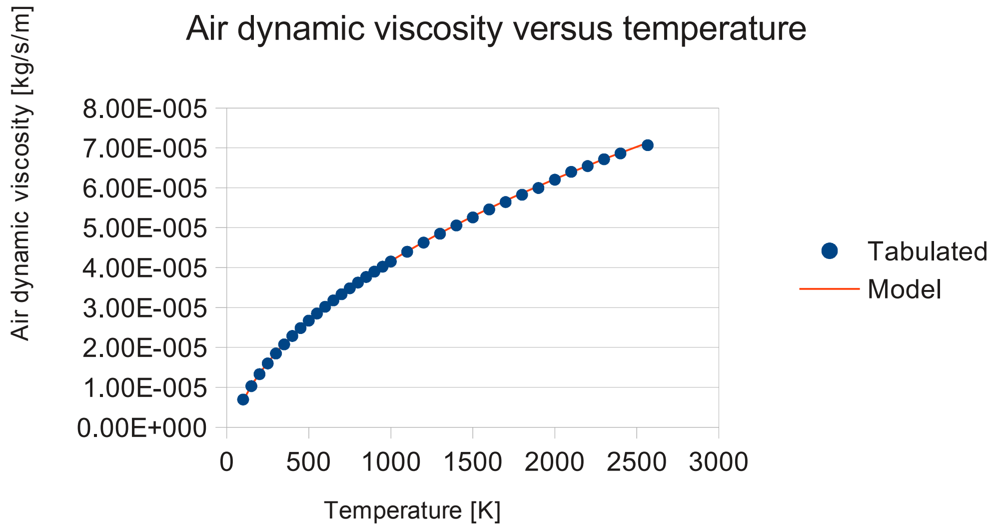


$$\rho(T) = \frac{360}{T} - \frac{0.114}{\sqrt{T}} \qquad 100\,K \le T \le 2500\,K$$

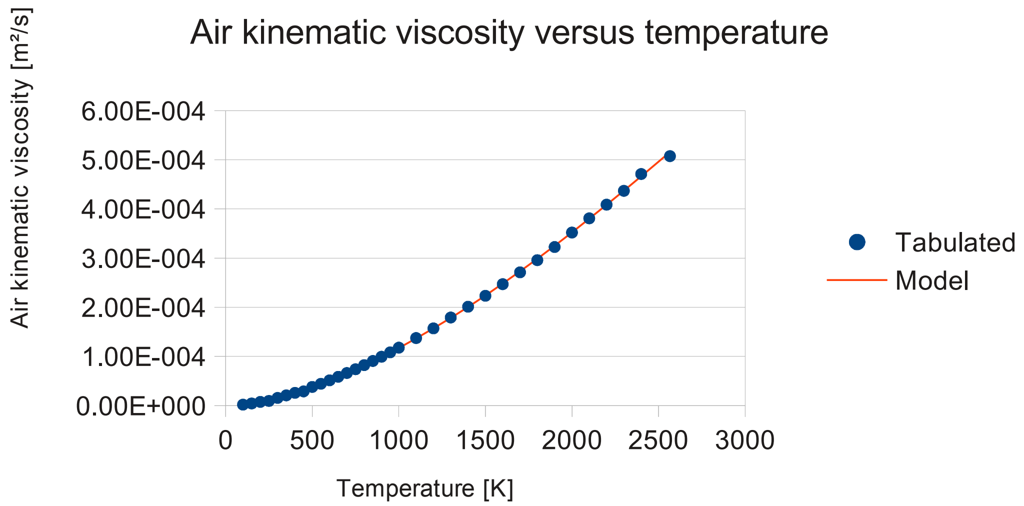
$$C_p(T) = 1030 - 0.24\,T + 6.85 \cdot 10^{-4}\,t^2 - 4.33 \cdot 10^{-7}\,T^3 + 9.45 \cdot 10^{-11}\,T^4 \qquad 100\,K \le T \le 2500\,K$$



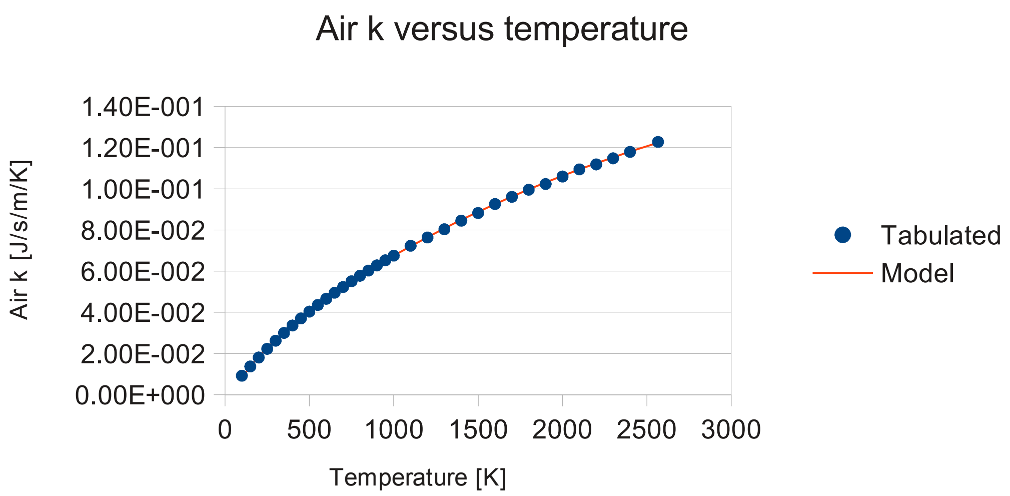
$$\mu(T) = -1.00\cdot10^{-5} - 1.47\cdot10^{-9}\,T + 1.68\cdot10^{-6}\sqrt{T} \qquad 100\,K \le T \le 2500\,K$$



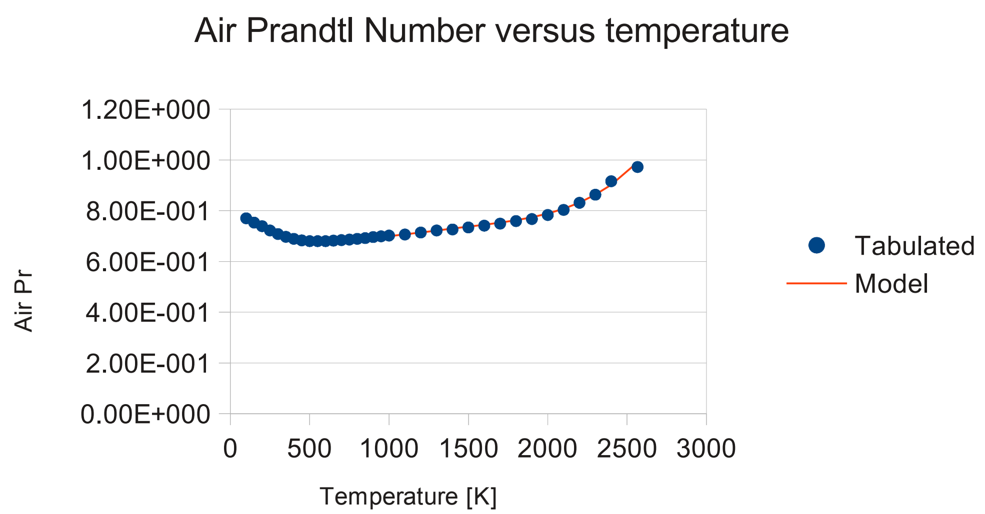
$$\nu(T) = -7.24\cdot10^{-6} + 5.30\cdot10^{-8}\,T + 7.95\cdot10^{-11}\,T^2 - 8.07\cdot10^{-15}\,T^3 \qquad 100\,K \le T \le 2500\,K$$



$$k(T)=-1.29\cdot10^{-2}+2.43\cdot10^{-5}T-3.39\cdot10^{-9}T^2+1.88\cdot10^{-3}\sqrt{T} \qquad 100\,K\leq T\leq 2500\,K$$

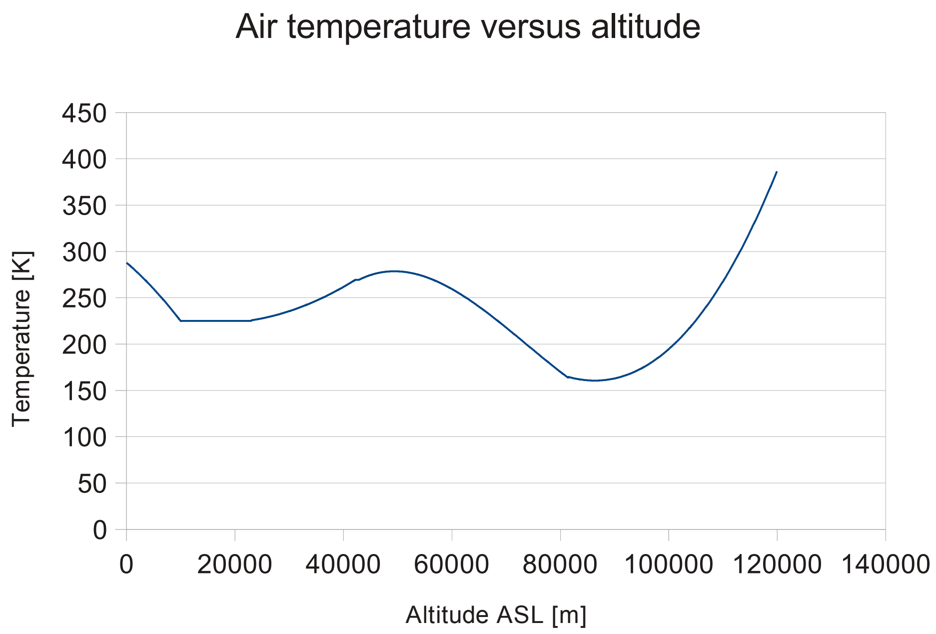


$$Pr(T)=0.815-5.31\cdot10^{-4}T+7.13\cdot10^{-7}T^2-3.69\cdot10^{-10}T^3+7.10\cdot10^{-14}T^4 \qquad 100\,K\leq T\leq 2500\,K$$



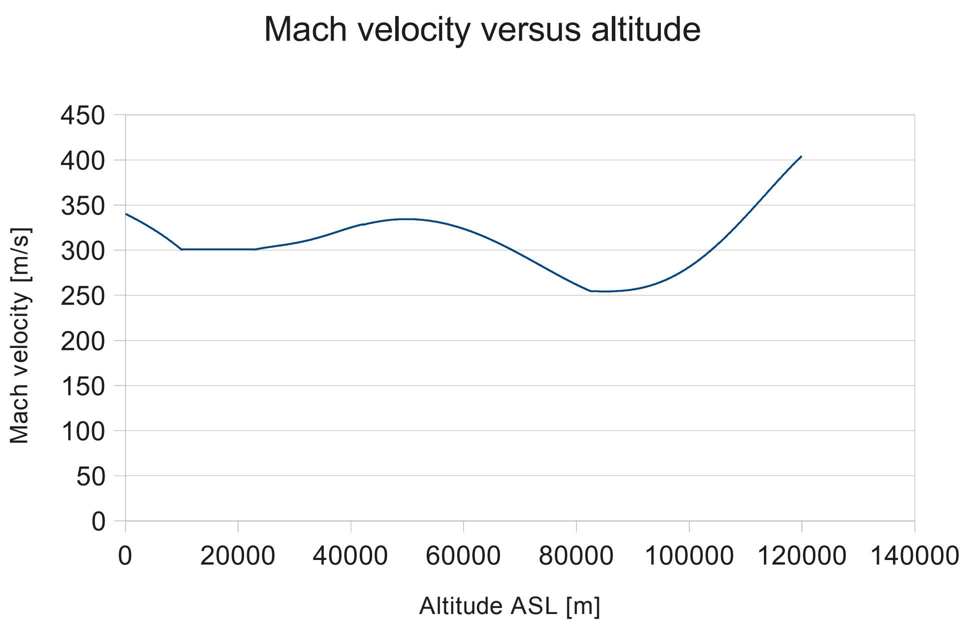
The air temperature T [K] versus altitude s [m] is curve fitted from [7]:

$$T[s]=\begin{cases}287.954-5.03015\cdot10^{-3}s-1.2859\cdot10^{-7}s^2 & 0\text{m}\leq s\leq 10.0\text{km}\\ 225.15 & 10\text{km}<s\leq 23\text{km}\\ 242.057-2.33854\cdot10^{-3}s+7.08133\cdot10^{-8}s^2 & 23\text{km}<s\leq 42\text{km}\\ -534.104+3.95468\cdot10^{-2}s-6.0177\cdot10^{-7}s^2+2.71838\cdot10^{-12}s^3 & 42\text{km}<s\leq 81.5\text{km}\\ 867.12-9.78603\cdot10^{-3}s-5.75164\cdot10^{-8}s^2+8.81316\cdot10^{-13}s^3 & 81.5\text{km}\leq s<120\text{km}\end{cases}$$



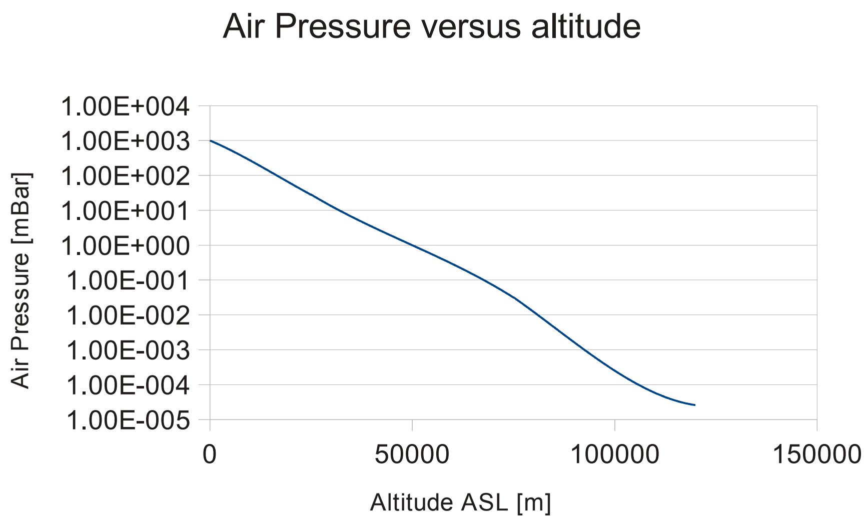
The free stream Mach velocity c [m/s] versus altitude s [m] is curve fitted from [7]:

$$c[s]=\begin{cases} 340.234-3.05528\cdot 10^{-3}s-6.35237\cdot 10^{-8}s^2-2.91745\cdot 10^{-12}s^3 & 0\text{m}\le s\le 10\text{km}\\ 300.8 & 10\text{km}<s\le 23\text{km}\\ -97.435+5.121\cdot 10^{-2}s-2.469\cdot 10^{-6}s^2+5.269\cdot 10^{-11}s^3-4.089\cdot 10^{-16}s^4 & 23\text{km}<s\le 42\text{km}\\ 287.23-6.725\cdot 10^{-3}s+4.103\cdot 10^{-7}s^2-6.824\cdot 10^{-12}s^3+3.371\cdot 10^{-17}s^4 & 42\text{km}<s\le 82\text{km}\\ -8698.8+3.91\mathrm{cdot}\,10^{-1}s-6.29\cdot 10^{-6}s^2+4.41\mathrm{cdot}\,10^{-11}s^3-1.13\cdot 10^{-16}s^4 & 82\text{km}\le s<120\text{km} \end{cases}$$



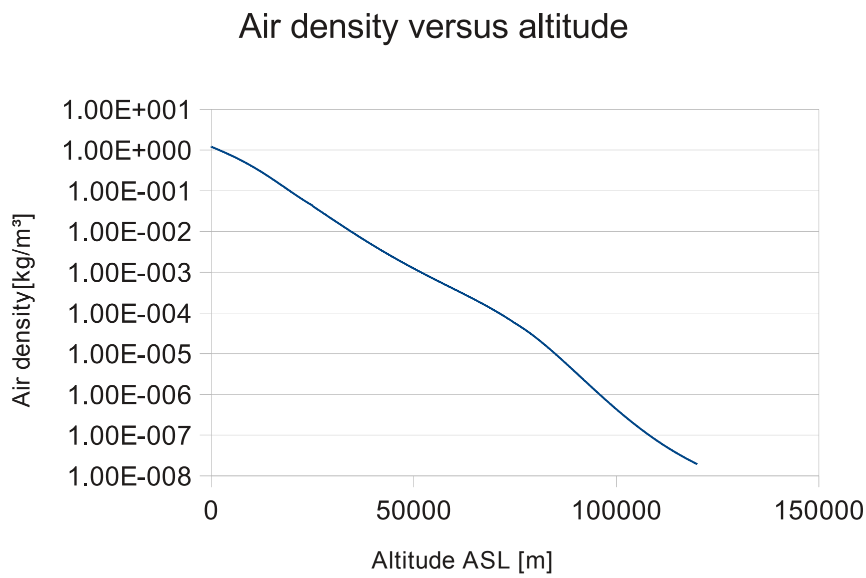
The air pressure P [mBar] versus altitude s [m] is curve fitted from [7]

$$P[s]=\begin{cases} \exp\left(4.43165\cdot10^{-14}s^3-2.28553\cdot10^{-9}s^2-1.14097\cdot10^{-4}s+6.91509\right) & 0\text{m}\le s\le 25\text{km} \\ \exp\left(-2.28179\cdot10^{-14}s^3+3.34063\cdot10^{-9}s^2-2.84655\cdot10^{-4}s+8.73033\right) & 25\text{km}< s\le 75\text{km} \\ \exp\left(4.44813\cdot10^{-14}s^3-1.13434\cdot10^{-8}s^2+7.62651\cdot10^{-4}s-15.5981\right) & 75\text{km}< s\le 120\text{km} \end{cases}$$



The air density ρ [kg/m³] versus altitude s [m] is curve fitted from [7]

$$\rho[s]=\begin{cases} \exp\left(4.88158\cdot10^{-18}s^4-1.808\cdot10^{-13}s^3+2.432\cdot10^{-11}s^2-9.693\cdot10^{-5}s+0.1922\right) & 0\text{m}\le s\le 25\text{km} \\ \exp\left(-6.034\cdot10^{-19}+1.035\cdot10^{-13}-5.746\cdot10^{-9}s^2-2.21\cdot10^{-5}s-0.396\right) & 25\text{km}< s\le 75\text{km} \\ \exp\left(-1.004\cdot10^{-18}s^4+4.440\cdot10^{-13}s^3-7.137\cdot10^{-8}s^2+4.773\cdot10^{-3}s-121.84\right) & 75\text{km}< s\le 120\text{km} \end{cases}$$



Properties behind a normal shock

The atmosphere properties behind a normal shock relates to the free stream values in the following way:

$\frac{P_0}{P_{fs}}=\frac{2\gamma M_{fs}^2-(\gamma-1)}{\gamma+1}$	Static pressure
$\frac{T_0}{T_{fs}}=\frac{[2\gamma M_{fs}^2-(\gamma-1)][(\gamma-1)M_{fs}{}^2+2]}{(\gamma+1)^2 M_{fs}{}^2}$	Static temperature
$\frac{\rho_0}{\rho_{fs}}=\frac{(\gamma+1)M_{fs}{}^2}{(\gamma-1)M_{fs}{}^2+2}$	Density
$\frac{P_{t0}}{P_{tfs}}=\left[\frac{(\gamma+1)M_{fs}{}^2}{(\gamma-1)M_{fs}{}^2+2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{(\gamma+1)}{2\gamma M_{fs}{}^2-(\gamma-1)}\right]^{\frac{1}{\gamma-1}}$	Total pressure
$\frac{T_{t0}}{T_{tfs}}=1$	Total temperature
$M_0=\frac{(\gamma-1)M_{fs}^2+2}{2\gamma M_{fs}{}^2-(\gamma-1)}$	Mach number