

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Fall 2007

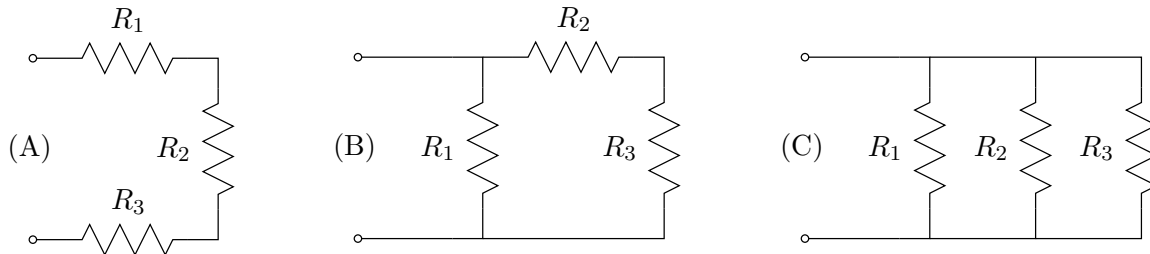
Problem Set #1

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Helpful readings for this homework: Chapter 1, Chapter 2.1-2.5

Exercise 1.0: Problem 1.4 makes use of WebLab, a physical remote-access laboratory that runs through the MIT iLab Service Broker. To obtain an iLab account, go to <http://ilab.mit.edu>, click on the *register here* link, provide the requested information, group yourself with Fall 2007 6.002 Students, and submit your request for an account. Account approval may take several hours to a day, so register early to avoid any last minute delays.

Exercise 1.1: Find the equivalent resistance, as viewed from its port, of each resistor network shown below.



Exercise 1.2: What is the largest-valued resistor that can be fabricated by combining a 3- Ω resistor, another 3- Ω resistor, and a 6- Ω resistor? What is the smallest-valued resistor that can be fabricated by combining the same three resistors? How can one fabricate a 5- Ω resistor by combining the same three resistors? Suppose that each individual resistor can dissipate up to 1 W of power before burning up. How much power can the fabricated 5- Ω resistor dissipate before burning up?

Problem 1.1: The network shown below in Figure 1 has four elements: two resistors, a current source and a voltage source. The values of the resistors and the magnitudes of the sources are all given. Branch currents (i_k) and voltages (v_k) are also defined for each element.

- (A) How many nodes are there in the network? Write a KCL equation for each node in terms of the branch currents i_k . How many of the KCL equations are independent?
- (B) How many loops are there in the network? (For the purposes of this problem, a loop is any direction-independent closed path through the network that passes through each branch no more than once.) Write a KVL equation for each loop in terms of the branch voltages v_k . How many of the KVL equations are independent?

- (C) Write an expression for the v - i constitutive law for each element.
- (D) By combining the independent equations from Parts (A) and (B) with the equations from Part (C), you should have a set of eight linear equations, matching in number the set of i_k plus v_k . Solve the equations to find all four i_k and all four v_k . Summarize your findings in a table.
- (E) Find all four branch powers $v_k i_k$. Show that the sum of the four $v_k i_k$ is zero, and hence that power is conserved in the network. Which branch elements source power and which branch elements sink power?

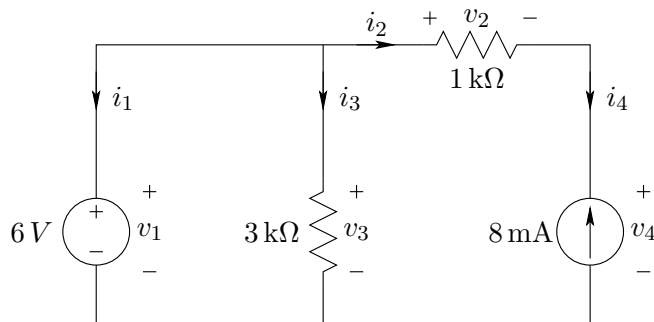


Figure 1: Network for Problems 1 and 2.

Problem 1.2: Replace the 8-mA current source in Problem 1.1 with a 2 k Ω resistor. For this new network, determine all branch voltages and all branch currents. However, rather than using the eight-equations-in-eight-unknowns approach used in Problem 1.1, make use of series and parallel resistor combinations, such as explored in Exercise 1.1, and voltage and current dividers.

Problem 1.3: One of the reasons that circuit analysis is so powerful is that it can provide a method for analogically studying a diverse range of problems outside of the field of electronics. This problem will explore the application of circuit theory to studying steady-state heat transfer through the wall of a typical residential building.

Thermal energy is transferred through the wall of a house via two different modes of heat transfer: conduction and convection. *Conductive heat transfer* occurs when thermal energy is transferred by direct contact between two solids, while *convective heat transfer* describes the transfer of thermal energy between a solid and an adjacent liquid or gas. One model of the manner in which both of these modes of heat transfer affect the temperature in a house is illustrated in Figure 2, where thermal energy is transferred from the outdoors (which is at a temperature T_{out}) to the external surface of the wall of the house by convection, the thermal energy is transferred from the outer surface of the wall to the inner surface through conduction, and then from the inner surface of the wall to the indoor air (at temperature T_{in}) again through convection.

One-dimensional conductive heat transfer through a material can be accurately modelled by Fourier's law:

$$\frac{Q}{A} = k \frac{\Delta T}{\Delta x}, \quad (1)$$

where Q is in units of Watts, A is in m^2 , k is in W/mK , T is in Kelvin, and x is in meters. This equation states that the transfer of thermal energy per unit area (otherwise referred to as heat flux, in W/m^2), is proportional to the temperature difference across the material, where k is a property

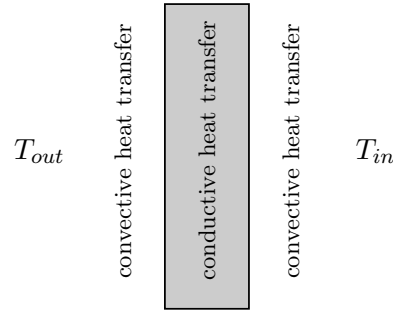


Figure 2: Heat flow through a wall.

of the material. It is important, in this formulation of Fourier’s law, to define ΔT such that it is positive; otherwise, the answer will assume that thermal energy flows from a body at a lower temperature to one at a higher temperature: this is quite unlikely!

Similarly, one-dimensional convective heat transfer between a fluid and a solid can be modelled by the following relation:

$$\frac{Q}{A} = \bar{h}(\Delta T), \quad (2)$$

where \bar{h} is the coefficient of average convective heat transfer, and is in $\text{W}/\text{m}^2\text{K}$.

- (A) One can construct an analogy between heat flow Q and current I , and between temperature T and voltage V . Reformulate both of the heat transfer relations defined above so that we can use Ohm’s law to construct “thermal” circuits; in other words, define equivalent thermal resistances R_{cond} and R_{conv} for the above heat transfer relationships by using the variables defined in equations 1 and 2.
- (B) Using your new definitions of thermal resistances, draw a thermal circuit model of the wall of Figure 3a, assuming that the wall has a thickness L and a surface area A , the thermal conductivity for the insulating wall material is k_i , the convective heat transfer coefficient is \bar{h} , the outside temperature is T_O , and that the inside temperature is T_I .

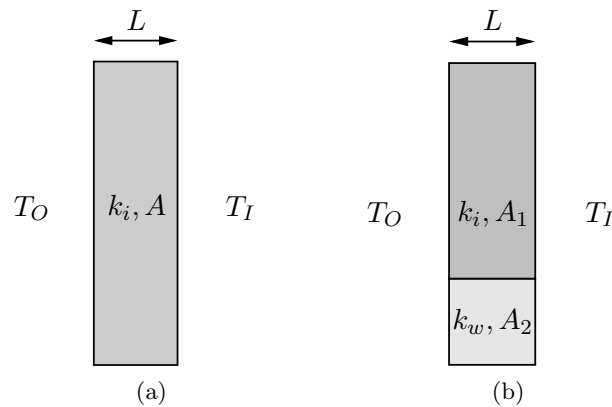


Figure 3: Two different thermal models of a wall.

- (C) Consider a more accurate model of a wall, illustrated in Figure 3b, in which the wall is constructed from wooden joists and insulation. Construct a thermal circuit model of this wall corresponding to the model shown in the picture, assuming that the insulation has surface area A_1 and thermal conductivity k_i , while the wood joists have net surface area A_2 and thermal conductivity k_w .

Now that you have some models of different configurations of the wall, we are going to evaluate these models and see what the differences in their behavior are. Representative numbers for the thermal conductivity of building materials are $k_i = 0.04\text{W/mK}$ for fiberglass insulation, and $k_w = 0.14\text{W/mK}$ for pine boards. In addition, assume that we are interested in evaluating a 1 m^2 section of 10 cm thick wall during the dead of winter, in which the indoor temperature is 25°C , and the outside temperature is -10°C . In such conditions, the indoor convective heat transfer coefficient is approximately $5\text{W/m}^2\text{K}$, while the outdoor convective heat transfer coefficient is $30\text{W/m}^2\text{K}$. Finally, assume for the model shown in Figure 3b that the surface area of the pine boards facing the external surface is 0.2 m^2 , while the surface area of the insulation is 0.8 m^2 .

- (D) For both of the thermal circuit models, calculate the temperature of the inside surface of the wall $T_{w,i}$, as well as the amount of heat transferred through the wall Q_{loss} . Compare these numbers; which wall construction would you prefer? Which model do you think might more accurately characterize a wall in your house, and how would you change the construction of the wall to improve the performance of either of the models?

Problem 1.4: This problem makes use of WebLab, a remote-access laboratory that enables experimentation controlled through a Java-enabled web browser. The remote laboratory, actually located in Building 38 at MIT, offers access to more expensive and sophisticated equipment than is available in the 6.002 laboratory. For example, this problem uses an HP4155B Semiconductor Parameter Analyzer to measure the voltage-current relation of a real diode as follows.

- Start WebLab
 - Go to <http://ilab.mit.edu> and login to your account on the MIT iLab Service Broker. If your account does not open up under the *My Labs* section of iLab, move to that section by clicking on the *My Labs* tab on the top bar. If you have trouble running iLab on one computer, try switching to another computer, or try moving to an Athena workstation. You can request help and advice from iLab personnel, or report a bug, by clicking on the Help tab on the top bar and using the Help Request form. This is the quickest way to get help; if the page is not available, you may also send mail to ilab-debug@mit.edu.
 - Select the *Microelectronics WebLab Graphical Client* by clicking on its link. Afterwards, you can click on the *Documentation* link for detailed information about the WebLab client.
 - Start the WebLab client by clicking the *Launch Lab* link.
- Set Up Experiment

Experiments are run through the WebLab window that will pop up following the launch of the WebLab client. In this experiment, you will measure the voltage-current relation of a diode.

- From the drop-down *Devices* menu on the top bar, select the (*p-n diode*). For this problem set, the diode is connected to the HP4155B through two Source Measurement Units

(SMU). SMU #1 is connected to the positive terminal of the diode, and SMU #2 is connected to the negative terminal of the diode. Each SMU can source either the voltage across, or the current through, its terminals. It can also measure both its current and voltage.

- Click on SMU #1. Name its current and voltage, and select the download option for both variables. Select a voltage sweep (*Mode = V & Function = VAR1*) from -1.5 V to 1.5 V with a 10-mV step. Set the compliance, or current limit, to 10 mA. Click on *Apply* and *OK* to complete the setup of SMU #1.
- Click on SMU #2. Set this SMU to ground (*Mode = COMM*). You must also name the voltage and current, but do not select them for downloading. (The voltage will always be zero, and the current will be the opposite of that measured by SMU #1. Thus, neither variable is interesting.) Click on *Apply* and *OK* to complete the setup of SMU #2.
- Run Experiment & View Measurements
 - From the drop-down *Measurement* menu on the top bar, select *Run Measurement*. Your experiment will be queued, and run on a first-come-first-served basis. Hint: if you wait until the last minute like everyone else, you may find the queue and the wait longer than desired.
 - The measured data will be displayed on a graph at the bottom of the WebLab window. It is probably best viewed with a linear X-axis display. Both a linear and a logarithmic Y-axis display are useful for answering the questions below. Note that the logarithmic display first takes the absolute value of the data.
 - Note the temperature of the diode. It is shown just above the measurement graph.
 - If for any reason you wish to clear your data and run the experiment again, you can first clear the data by selecting *Clear Data* from the *Results* drop-down menu on the top bar.
- Download Measurements

To download the measured voltage and current data to your local computer, select *Download Data* from the *Results* drop-down menu on the top bar.

Use your voltage and current data measured for the diode via WebLab to answer the following questions. As you view the data, remember that SMU #1 was set up to limit the diode current to 10 mA, even if the applied voltage attempts to drive a larger current.

(A) In theory, the voltage-current (v_D - i_D) relation for a diode is

$$i_D = I_S(e^{v_D/V_{TH}} - 1) .$$

The thermal voltage V_{TH} is given by $V_{TH} = kT/q$, where $k = 1.38 * 10^{-23}$ J/K is Boltzmann's constant, T is the diode temperature in degrees Kelvin, and $q = 1.60 * 10^{-19}$ C is the electron charge. Given the measured diode temperature, compute V_{TH} .

(B) Over what range of voltage does the measured diode data exhibit $i_D \approx -I_S$? Is this as expected from the theoretical relation given above? Using the measured data, estimate I_S .

- (C) Over what range of voltage does the measured diode data exhibit the exponential relation $i_D \approx I_S e^{V_D/V_{TH}}$? Is this as expected from the theoretical relation given above?

The following hints may help in answering Parts (B) and (C) above.

- Parts (B) and (C) can be answered either graphically or numerically. If you choose a graphical method, then you should plot the downloaded data using MatLab, Excel or another program, or print a screen-shot of the graph produced by WebLab. Capturing screen-shots is a platform specific activity; consult the WebLab documentation for Windows- and Athena-specific advice. If you choose a numerical method, then you could process the downloaded data using MatLab, Excel or another program.
- Before answering Parts (B) and (C), you may find it instructive to examine the measured data visually using WebLab. In this case, a logarithmic y-axis display is probably best used for Parts (B) and (C).