Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Fall 2007

Problem Set 10

Issued 11/13/2007 – Due 11/27/2007

Helpful readings for this homework: Chapter 14

Exercise 10.1: Exercise 14.4 from Chapter 14 of A&L (page 824). Hint: Use the impedance method.

Exercise 10.2: For each of the filters shown in Figure 1 determine whether it is a low pass filter (LPF), a high pass filter (HPF), a band pass filter (BPF), or a band stop filter (BSF – A band stop filter passes low and high frequencies, but attenuates frequencies in between).
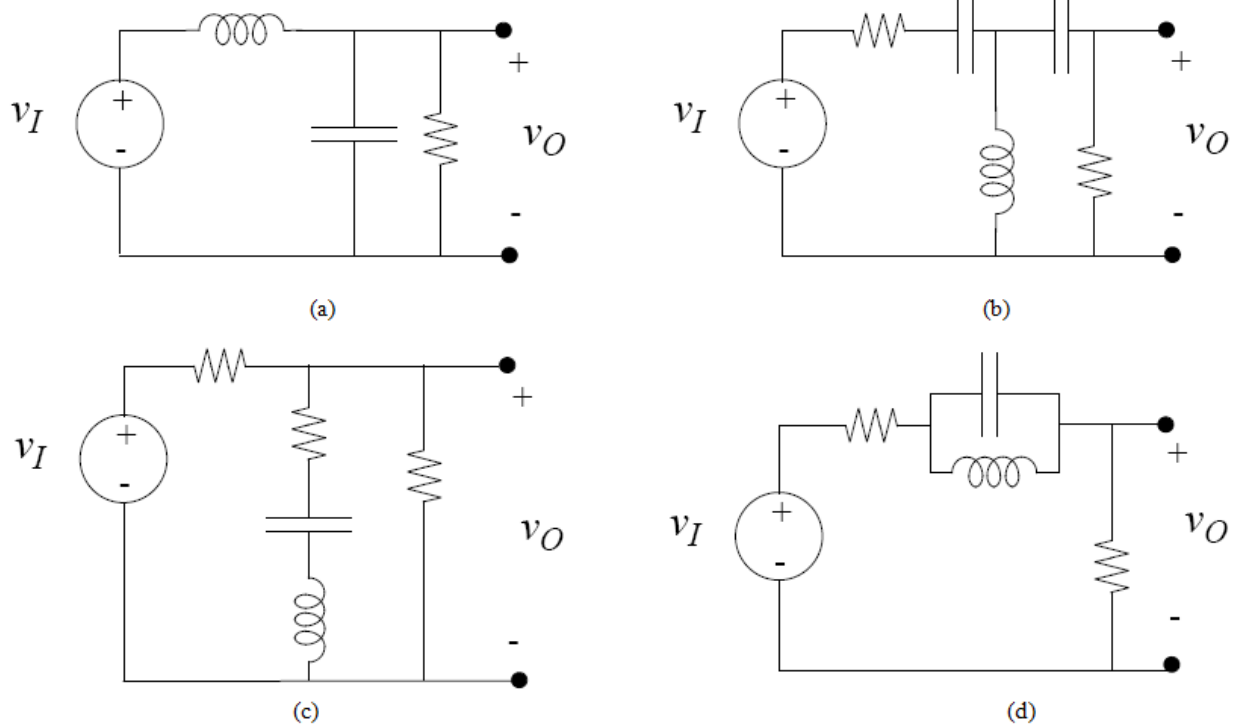

Figure 1

Problem 10.1: The network shown in Figure 2 is driven in steady-state by the sinusoidal current $i_{IN}(t) = I_{in}\cos(\omega t)$. The output of the network is the voltage $v_{OUT}(t)$, which takes the form $v_{OUT}(t) = V_{out}\cos(\omega t + \phi)$. Find V_{out} and ϕ as functions of ω as follows.

(A) Find a differential equation that can be solved for $v_{out}(t)$ given $i_{IN}(t)$. Hint: consider how $v_{OUT}(t)$ is related to the inductor current.

(B) Let $i_{IN}(t) = Re\{\hat{I}_{in}e^{j\omega t}\}$. Also let $v_{OUT}(t) = Re\{\hat{V}_{out}e^{j\omega t}\}$, where $\hat{V}_{out}$ is a complex function of the circuit parameters, ω and I_{in}. With these definitions, find $\hat{V}_{out}$.

(C) An alternative way to write $v_{OUT}(t)$ is as $v_{OUT}(t) = Re\{|\hat{V}_{out}|e^{j(\omega t + \angle\hat{V}_{out})}\}$. Determine $|\hat{V}_{out}|$ and $\angle\hat{V}_{out}$ as functions of the circuit parameters, ω and I_{in}. Then, find V_{out} and ϕ for the original cosine input, again both as functions of the circuit parameters ω and I_{in}.

(D) Sketch and clearly label V_{out}/I_{in} and ϕ as functions of ω. Identify the low-frequency and high-frequency asymptotes on the sketch.
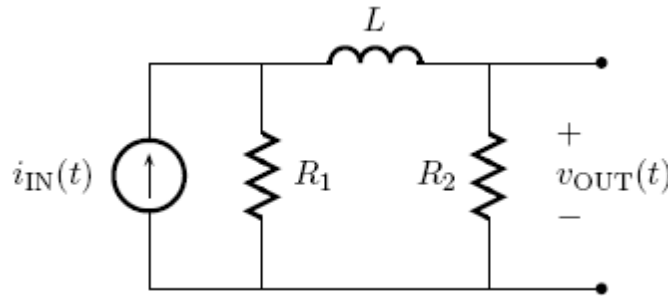

Figure 2

Problem 10.2: This problem concerns the sinusoidal-steady-state behavior of the two networks shown below, both of which have two ports.
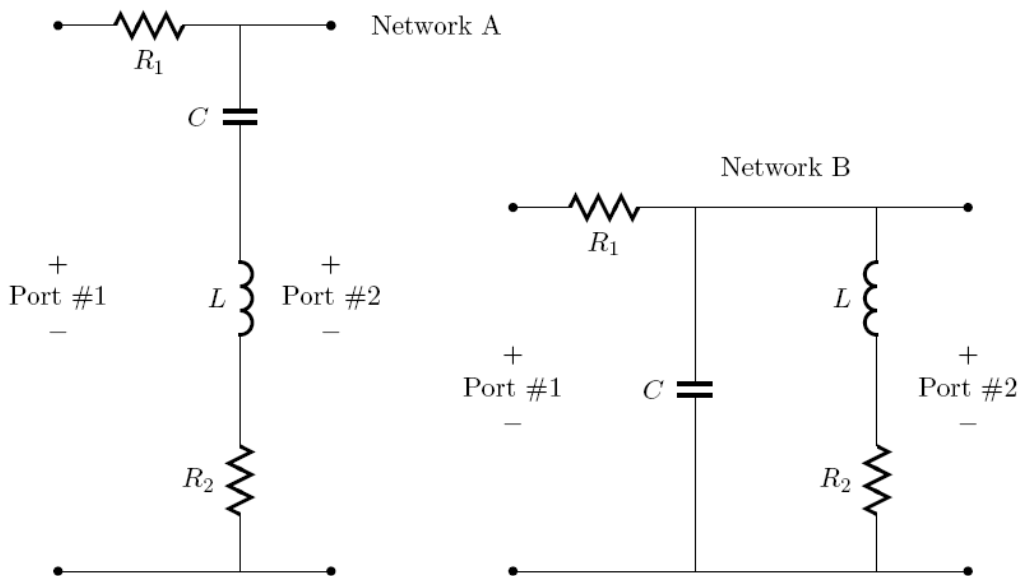

Figure 3

2

(A) Determine the impedance of each network as viewed into Port #1 under the assumption that Port #2 is open.
(B) Assume that Port #1 of each network is driven in sinusoidal steady state by the voltage $V_1 cos(\omega t)$, and that Port #2 is open. Determine the current into the positive terminal of each network at Port #1. Express the current in the form $I_1 cos(\omega t + \phi_1)$ where I_1 is an amplitude and ϕ_1 is a phase angle.
(C) Assume that Port #1 of each network is again driven in sinusoidal steady state by the voltage $V_1 cos(\omega t)$, and that Port #2 is again open. Determine the voltage that appears at Port #2. Express the voltage in the form $V_2 cos(\omega t + \phi_2)$ where V_2 is an amplitude and ϕ_2 is a phase angle.
(D) Finally, use WebLab to study Network A, which is a notch filter. Note that R_2 actually models the winding resistance of the inductor. To study Network A, start the iLab Service Broker at <iLab.mit.edu>, select the Dynamic Signal Analyzer Lab Client, and launch the lab. Finally, run the WebLab client to measure the frequency response of the filter using the standard parameters of a 2-V peak-to-peak input, a 1-Hz start frequency and a 50-kHz stop frequency. Based on the measured response, and the knowledge that L = 98 mH, estimate R_1, R_2 and C.

Problem 10.3: This problem examines the very simple tuner for an AM radio shown below. Here, the tuner is the parallel inductor and capacitor. The injection of radio signals into the tuner by the antenna is modeled by a current source, while the Norton resistance of the antenna in parallel with the remainder of the radio is modeled by a resistor. (You will learn about antenna modeling in 6.013.) The AM radio band extends from 520 kHz through 1610 kHz. The information transmitted by each radio station is constrained to be within ±5 kHz of its center frequency. (You will learn about AM radio transmission in 6.003.) To prevent frequency overlap of neighboring stations, the center frequency of each station is constrained to be a multiple of 10 kHz. Therefore, the purpose of the tuner is to pass all frequencies within 5 kHz of the center frequency of the selected station, while attenuating all other frequencies.
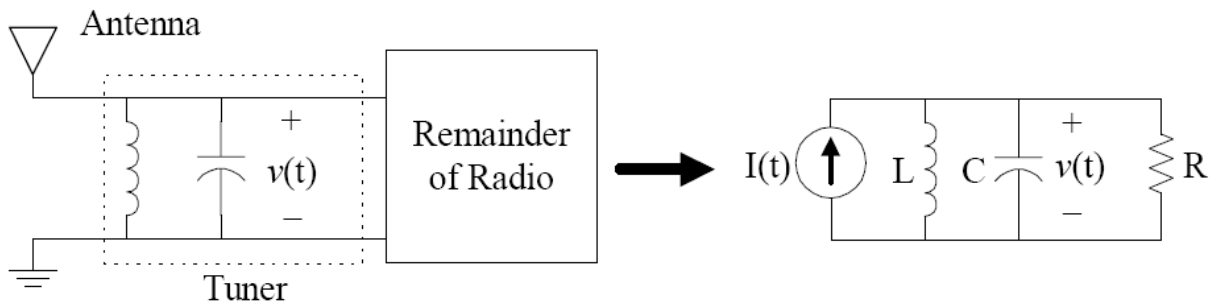

Figure 4

(A) Assume that $I(t) = I cos(\omega t)$. Find $V(\omega)$ and $\phi(\omega)$, where $v(t) = V cos(\omega t + \phi)$. Note that $v(t)$ is the output of the tuner, namely the signal that is passed on to the remainder of the radio.
(B) For a given combination of I, C, L and R, at what frequency is V maximized?
(C) Assume that $L = 470$ μH. Over what range of capacitance must C vary so that the frequency of maximum V/I may be tuned over the entire AM band. Note that tuning the

frequency of maximum V/I to the center frequency of a particular station tunes in that station.

(D) As a compromise between passing all frequencies within 5 kHz of a center frequency and rejecting all frequencies outside that band, let the design of R be such that V (1 MHz $\pm$ 5 kHz)/V (1 MHz) ≈ 0.25 when the tuner is tuned to 1 MHz. Given this design criterion, determine R.

(E) Given your design for R, determine V(1 MHz $\pm$ 10kHz)/V(1 MHz). Also, determine Q for the tuner and its load resistor when the tuner is tuned to 1 MHz.

(F) Suppose the tuner is first tuned to another station and then quickly tuned to the station broadcasting at 1 MHz. Approximately how long will it take for $v(t)$ to switch to the station broadcasting at 1 MHz. Assume that both stations broadcast signals of equal strength. Hint: consider the time-domain interpretation of Q.