

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Fall 2007

Homework #8
Issued 10/30/2007 – Due 11/7/2007

Reading: Chapter 10 and Chapter 12

Exercise 8.1: Chapter 10, Exercise 10.16 (p. 572).

Exercise 8.2: Chapter 12, Exercise 12.3 (p. 695).

Problem 8.1: An RL circuit is shown below:

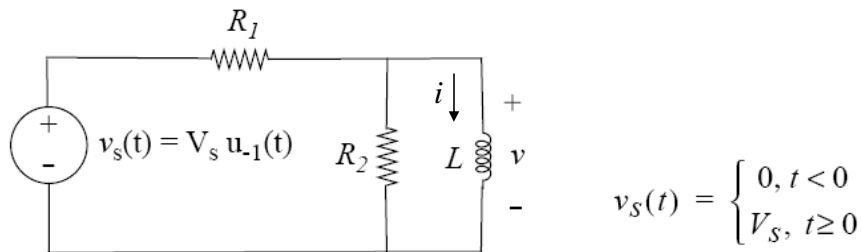


Fig.1 RL circuit of problem 8.1.

The current in the inductor is zero for $t < 0$.

- a) Write a differential equation for $v(t)$ and $i(t)$ and solve them for $t > 0$. Sketch and dimension $v(t)$.
- b) The input is changed to an impulse of magnitude (area) Λ . That is: $v_s(t) = \Lambda u_0(t)$. Without solving the differential equation, derive an expression for $v(t)$ when $t > 0$. Sketch and dimension the result.
- c) The input is changed to a ramp of magnitude (slope) M , as shown in figure 2. Without solving the differential equation again, derive an expression for $v(t)$ for $t > 0$. Sketch and dimension the result.
- d) The input is changed to that shown in figure 3. Show that this input can be decomposed into two ramps and two steps.
- e) Using your results from a) and c) above, by superposing response to step and ramp inputs, derive an expression for $v(t)$ when $t > 0$.

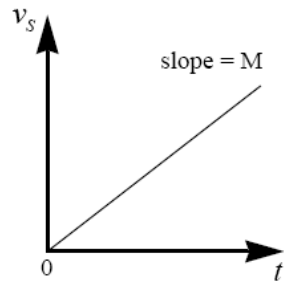


Fig.2 Ramp input in part c).

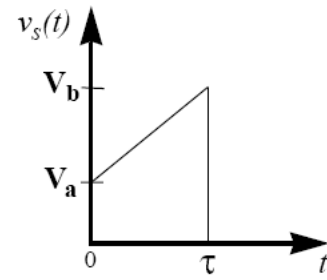


Fig.3 Combined input in part d).

Problem 8.2: This problem involves studying theoretically and experimentally the step response of a series RC circuit. The experiments are to be done using the ELVIS station from National Instruments that is available through the iLab website. Login into the site and select “ELVIS Lab Client” as your Lab. When you launch the lab, you will see the circuit under test, which is reproduced below.

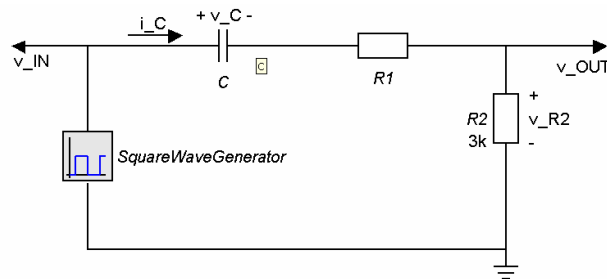


Fig.4 Experimental circuit in problem 8.3.

In this circuit, we have a capacitor and two resistors all in series with a function generator. Resistor R2 is a small 3Kohm resistor that is introduced to sample the current through the series network. We can measure the voltage across R2 using an oscilloscope.

First, let's carry out a theoretical analysis of this circuit. Consider that a periodic square-wave input is applied with an amplitude V and a period T that is much larger than the time constant of the RC network, as shown in Fig.5. The waveform was applied long before time $t=0$ while the measurements always start at time $t=0$. So we can assume that the circuit is at certain steady state at time $t=0^-$ just before the measurements start.

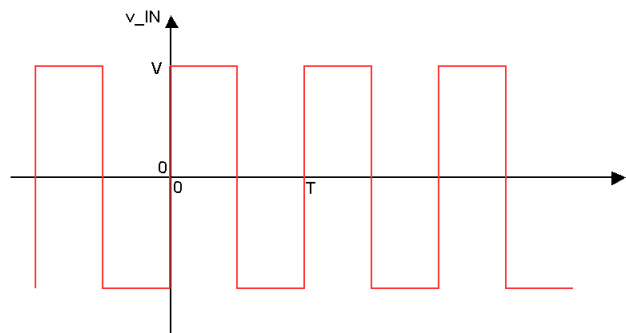


Fig.5 Square waveform used to stimulate the circuit

- a) Derive a time dependent expression for the waveform of the voltage across the capacitor $v_C(t)$. This should include separate expressions for the time period when the input is $v_I = V$ and when it is $v_I = -V$.
- b) Derive a time dependent expression for the waveform of the current flowing through this circuit $i_C(t)$.
- c) Derive a time dependent expression for the voltage across resistor R2, $v_{R2}(t)$.

Now let us carry out some measurements on the circuit that is available online.

First, select the signal generator (FGEN) and set its parameters to WaveForm = SQUARE, Frequency = 200 Hz, Amplitude = 1 V, and Offset = 0 V. Second, select the output measurement unit (SCOPE) and think about a suitable sampling rate that will allow you to see at least one full cycle of the output waveform with enough resolution. Note that the system will only allow you to take a maximum of 201 data samples at the output. Third, run the experiment. Finally, select v_{IN} for the Y1 axis and v_{OUT} for the Y2 axis, and use linear axes for both. When the figure resembles what you expect, capture a screen shot.

- d) From the data that you have obtained, extract the time constant of the RC circuit. You can see the actual numerical values of the data that you have obtained by looking into “View Data” under the “Results” menu. You can also download the data to Excel under the “Results” menu.
- e) From the data that you have obtained, extract the value of the resistor R1. When you do this, note that even though you selected 1V as the amplitude, the tool does not do this very accurately and the actual amplitude is being measured as VIN.
- f) From the data that you have obtained, extract the value of the capacitor C.

Problem 8.3: In the network shown below, the inductor and the capacitor have zero current and voltage, respectively, prior to $t = 0$. At $t = 0$, a step in voltage from 0 to V_o is applied by the voltage source indicated.

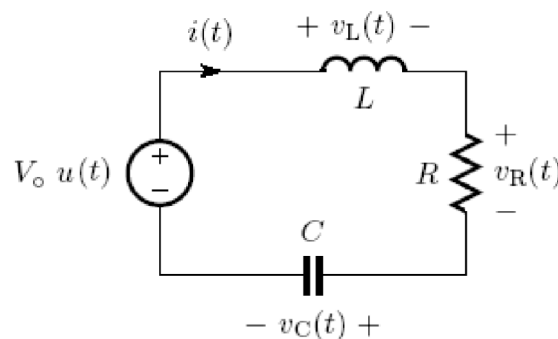


Figure 6: A step-driven series RLC circuit.

- a) Find v_C , v_L , v_R , i , di/dt just after the step at $t = 0^+$.
- b) Assuming $R = 0$, find an expression for $i(t)$ for $t > 0$.
- c) Now assuming that R has a nonzero value, find i at $t = \infty$.
- d) Find a second-order differential equation which describes the behavior of $i(t)$ for $t > 0$.
- e) Following parts (a), (b), and (c), derive an expression for current $i(t)$ takes in terms of V_o , R , L and C .